


# 15. Electromagnetic Induction

## Magnetic flux linked with a surface:

- It is the number of magnetic lines of force crossing a surface.
- Flux linked with a surface of area  $A$  placed in a magnetic field  $B$ ,  
capital  $\phi$  equals  $B$  with rightwards arrow on top.  $A$  with rightwards arrow on top equals space  $B$   $A$  space  $\cos$  space  $\theta$   
.
- SI unit of flux is Weber.

## Faraday's Laws

- First Law – Whenever the amount of magnetic flux linked with a circuit changes, an emf is induced in the circuit. The induced emf lasts as long as the change in magnetic flux continues.
- Second Law – The magnitude of emf induced in a circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.  
The emf induced in a coil for changing magnetic flux( $\phi_B$ ) linked with it is given by  
 equals negative  $N$  fraction numerator  $d$  capital  $\phi$  subscript  $B$  over denominator  $d t$  end fraction.

## Fleming's Right Hand Rule

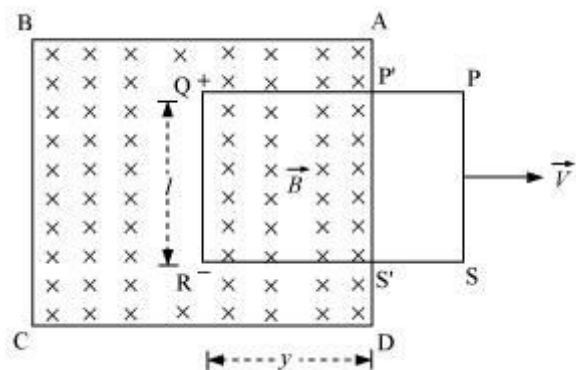
The thumb, the first finger and the middle finger of the right hand are stretched such that they are mutually perpendicular to each other. If the first finger is along the direction of the magnetic field and the thumb is along the direction of the motion of the conductor, then the middle finger represents the direction of induced emf or current in the conductor.

## Lenz's law

- It states that the direction of induced emf in a circuit is always such to oppose the change in magnetic flux responsible for it.
- It is in accordance with the principle of conservation of the energy.
- It is used to find the polarity of the emf induced in a circuit ,magnetic flux related to which is changing with time.
- The negative sign in the expression of the induced emf( $\epsilon = -\frac{d\Phi_B}{dt}$ ) is given by this law.

## Motional Electromotive Force





- For a conductor PS of length  $l$  under the following conditions:
  - moving with the velocity  $v$
  - in a magnetic field of strength  $B$  the
- magnetic flux linked with the circuit changes and the emf induced ( $e$ ) is given by  $e = -Blv$

### Energy consideration

- When a conductor is moved in a magnetic field there is conversion of the mechanical energy into electrical and thermal energy.
- For a rod of the resistance  $r$  the current in the circuit is given by  $i = Blv/r$ .
- The force that opposes the rod to move is given by  $F = ilB = B^2 l^2 v/r$ .
- Power required to move the rod in the magnetic field is given by  $P = B^2 l^2 v^2/r$
- Energy dissipated as heat is given by  $PJ = B^2 l^2 v^2 r$
- **Eddy currents** → Currents induced in a thick conductor when the conductor is placed in a changing magnetic field.

### Applications of Eddy Currents

- **Electromagnetic damping** – Some galvanometers have a fixed core, which is made of non-magnetic metallic materials. When the coil oscillates, the eddy currents generated in the core oppose the motion and bring the coil to rest quickly.
- **Induction furnace** – In an induction furnace, very high temperature can be produced by producing large eddy currents.
- **Magnetic braking in trains** – Strong electromagnets are situated above the rails in some electrically powered trains. When the electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train.

- **Self-induction** – Property of a coil by virtue of which the coil opposes any change in the strength of current flowing through it by inducing an *emf* in itself
- **Self-Inductance of a Long Solenoid of length  $l$**

$$L = \mu_0 N^2 A / l$$

$N$  is the number of turns in the coil

- **Mutual induction:** It is the phenomenon of production of induced emf in one coil, due to varying current in the neighbouring coil.

Induced emf,

$$E = -M \frac{dI}{dt}$$

Where,  $M$  is mutual inductance of the coil

- Mutual inductance of two long solenoids:

$$M = \frac{\mu_0 N_p N_s A}{l}$$

Where,

$N_p \rightarrow$  Number of turns in the primary coil

$N_s \rightarrow$  Number of turns in the secondary coil

$A \rightarrow$  Cross-sectional area of the solenoid

$l \rightarrow$  Length of the solenoid

## Displacement current

- It is the current that exists in the region where the electric field and the electric flux is changing with time.
- The displacement current is given by  $I_D = \epsilon_0 d\Phi E / dt$ 
  - where,  $\epsilon_0$  = absolute permittivity of free space,
  - $d\Phi E / dt$  = time rate of change of the flux.

## Ampere-Maxwell's Law

- According to this law the line integral of the magnetic field ( $\vec{B}$ ) over a closed path is equal to  $\mu_0$  times the sum of the conduction current ( $I$ ) and the displacement current ( $I_D$ )
- $\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \epsilon_0 d\Phi E / dt$
- In an **AC generator**, mechanical energy is converted into electrical energy by virtue of electromagnetic induction. If a coil of  $N$  turns and area  $A$  is rotated at  $n$  revolutions per second in a uniform magnetic field  $B$ , then the motional emf produced is

$$\mathcal{E} = NBA(2\pi\nu) \sin(2\pi\nu t)$$

Where, we have assumed that at  $t = 0$  s, the coil is perpendicular to the field.

## Alternating Current (AC)

Alternating current is an electric current whose magnitude changes with time continuously and which reverses its direction periodically. It is mathematically represented as  $I = I_0 \cos \omega t$  or  $I = I_0 \sin \omega t$

### Advantages of AC over DC

- Flexibility of converting from one value to other using transformer
- Can be transmitted over long distances economically as well as without much power loss

### Mean Value or Average value of AC ( $I_m$ or $V_m$ )

It is defined as that value of steady current which sends the same amount of charge through a circuit in the time of half cycle (i.e.  $T/2$ ) as is sent by AC through the same circuit in the same time. Let, in small time  $dt$ , charge sent in the circuit due to AC,  $I = I_0 \sin \omega t$ , is  $q$ .

$$I_m = 2\pi I_0 = 0.637 I_0$$

$$V_m = 2\pi V_0 = 0.637 V_0$$

### Root Mean Square Value (r.m.s.) of AC ( $I_v$ or $V_v$ )

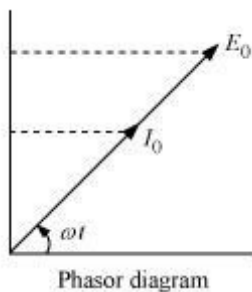
It is defined as that value of steady current which generates the same amount of heat in a resistor in a given time as is done by AC through the same resistor in the same time.

$$I_v = I_0 / \sqrt{2} = 0.707 I_0$$

$$V_v = V_0 / \sqrt{2} = 0.707 V_0$$

### AC through a resistor:

- When AC flows through a resistor, the voltage and current are in phase with each other.

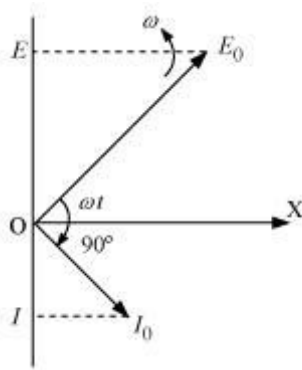


### AC through an inductor:

- The alternating emf is ahead of alternating current by a phase angle of  $\pi/2$ .

Inductive reactance ( $X_L$ ):

$$X_L = \omega L = 2\pi fL$$

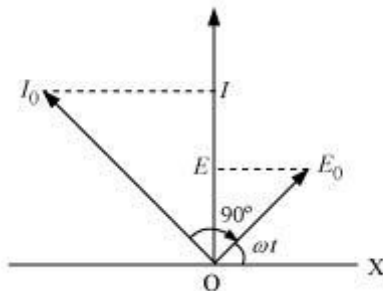


### AC through a capacitor:

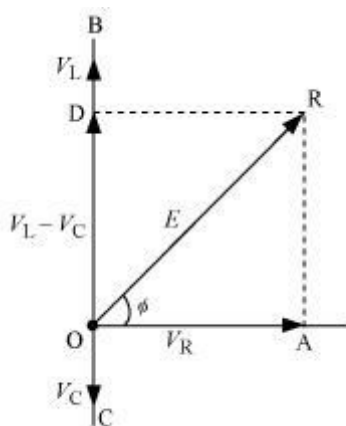
- The current leads the emf by a phase angle of  $\pi/2$ .

Capacitive reactance ( $X_C$ ):

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$



- AC through *LCR* series circuit:



$$E = \sqrt{I^2 R^2 + (X_L - X_C)^2}$$

$$\tan \Phi = \frac{X_L - X_C}{R}$$

Impedance ( $Z$ ):

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- **Power in LCR circuit:**

$$P_{av} = \frac{E V^2}{R^2 + \omega L^2 + \frac{1}{\omega C^2}}$$

$$\text{Power factor} = \cos \Phi = \frac{R}{\sqrt{R^2 + \omega L^2 + \frac{1}{\omega C^2}}}$$

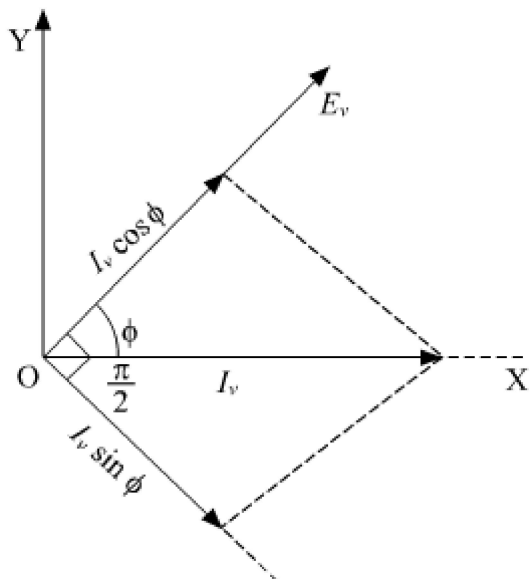
### Resonance

- It is the property exhibited by an LCR circuit.
- At a certain frequency known as resonating frequency ( $\omega_0$ ) the current through the circuit is maximum.
- It occurs at the frequency that can make  $X_L = X_C$  or  $\omega L = \frac{1}{\omega C}$ .
- Impedence of the circuit  $Z = R$ .
- resonating frequency  $\nu = \frac{1}{2\pi LC}$
- Quality factor,  $Q = \frac{1}{R} \sqrt{\frac{L}{C}}$  is used to measure the sharpness of the resonance.

### Power in AC circuit

- In AC circuit the power is given as  $P = VI \cos \phi$ 
  - where,  $V$  = rms value of the voltage
  - $I$  = rms value of the voltage
  - $\cos \phi$  = power factor =  $\frac{\text{true power}}{\text{apparent power}}$
  - $\Phi = \tan^{-1} \frac{X_C - X_L}{R}$
- Cases for power factor
  - For purely resistive circuit  $\cos \phi = 1$ ,  $\phi = 0$
  - For purely capacitive circuit  $\cos \phi = 0$ ,  $\phi = \frac{\pi}{2}$
  - For purely inductive circuit  $\cos \phi = 0$ ,  $\phi = -\frac{\pi}{2}$
- **Wattless Current or Idle Current**

It is that current which consumes no power for its maintenance in an electric circuit. In an inductive circuit, the current lags behind the voltage which is shown in the figure below.



The component  $I_v \sin \phi$  makes no contribution to the consumption of power in the a.c. circuit. Hence, it is known as wattless current.

### LC Oscillations

- When capacitor of capacitance  $C$  charged to and inductor of inductance  $L$  are connected then:
  - energy stored in  $C$  oscillates between  $L$  and  $C$ .
  - energy of the oscillations is given by  $v = \frac{1}{2\pi LC}$ .
  - Total energy in  $L$  and  $C$  every instant remains constant.
- A transformer consists of an iron core, on which are bound a primary coil of  $N_p$  turns and a secondary coil of  $N_s$  turns. If the primary coil is connected to an AC source, the primary and secondary voltages are related by

$$V_s = \left( \frac{N_s}{N_p} \right) V_p$$

And the currents are related by

$$I_s = \left( \frac{N_p}{N_s} \right) I_p$$

If  $N_s > N_p \rightarrow$  The voltage is stepped up (step-up transformer)

If  $N_s < N_p \rightarrow$  The voltage is stepped-down (step-down transformer)